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Subatomic physics

Final exam

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Exercise 1

Special relativity , four vectors

(a) Define mathematically what space-like, time-like and light-like events are, explain what they physically represent and draw one of each kind on a light cone in the $x-ct$ plane. **(1)**

(b) A particle has energy E and momentum p_x, p_y and p_z. Write down the general form of the covariant and contravariant energy-momentum form vectors. What object is used to make the transformation between the two forms? Write down its form. Work with natural units. **(1)**

(c) Is a real particle space-like, time-like or light-like and why? Similarly for a massless and a virtual particle. Explain why you make its association based on the relationship between the components of the four vector of each case. **(1)**

(d) Cosmic ray muons are produced high in the atmosphere (let's say at 10000 m) and travel with a velocity close to the speed of light (let's say $v = 0.999c$). The proper lifetime of the muon is $\tau = 2.2\ \mu$s.

1. How far does the muon travel according to its own reference frame? **(1)**
2. How far does the muon travel according to an observer at rest on Earth? **(1)**
3. Based on the two previous answers, does the muon reach the ground? Discuss if these two previous results are consistent and why. **(1)**

It is given that: $\gamma = 1/\sqrt{1-(v/c)^2} = 22.4$ and $c \approx 3 \cdot 10^8$ m/sec.

(e) A proton with mass $m_{\rm p}$ is accelerated to energy $E_{\rm p}$ and hits another proton at rest. Find the threshold energy for the reaction $\rm p + p \rightarrow p + p + \pi^+ + \pi^-$ to take place, considering that the masses of the negative and positive pions are $m_\pi = m_{\pi^-} = m_{\pi^+}$. **(1)**

Exercise 2

QED and weak interactions

(a) The QED Lagrangian can be derived from the free Dirac Lagrangian as a starting point. How many are the solutions of the equations of motion of the Dirac Lagrangian and what do they represent? **(1)**

(b) Write down the QED Lagrangian and explain what each term represents. **(1)**

(c) What kind of particles do the Ψ-symbols represent in the QED Lagrangian? **(1)**

(d) One of the well known QED processes is the elastic scattering of energetic electrons off protons at rest. The cross section of such process is given at the end by the so-call Rosenbluth formula. Write down its expression and explain what the G-terms physically represent. What do we learn from them? **(1)**

(e) What are the different types of weak processes that we can have based on the types of particles involved in the initial and final state of an interaction? Write down one characteristic interaction of each type. **(1)**

(f) One of the characteristic features of the weak sector of the Standard Model is embodied in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Which one is it? What does each element of the matrix represent? **(1)**

Exercise 3

Strong interactions

(a) Write down the form of the QCD Lagrangian in the Standard Model. Explain what each term represents. **(1)**

(b) The QCD Lagrangian looks quite similar to the QED one (see your answer to a previous question). Identify and discuss the differences between the two Lagrangians for every single term. **(1)**

(c) QCD has a number of unique features that the other theories of the Standard Model do not have. Asymptotic freedom and confinement are the most characteristic of them.

1. Explain what do they physically mean. **(1)**
2. and where do they originate from. **(1)**

(d) For $N_c = 3$ determine an upper limit for n_f for which QCD is still asymptotically free. **(1)**

(e) There are nine gluon species which are described by the SU(3) color symmetry $3 \otimes \bar{3} = 8 \oplus 1$. Why do we only consider 8 of them as physical? Why is the singlet state excluded? **(1)**

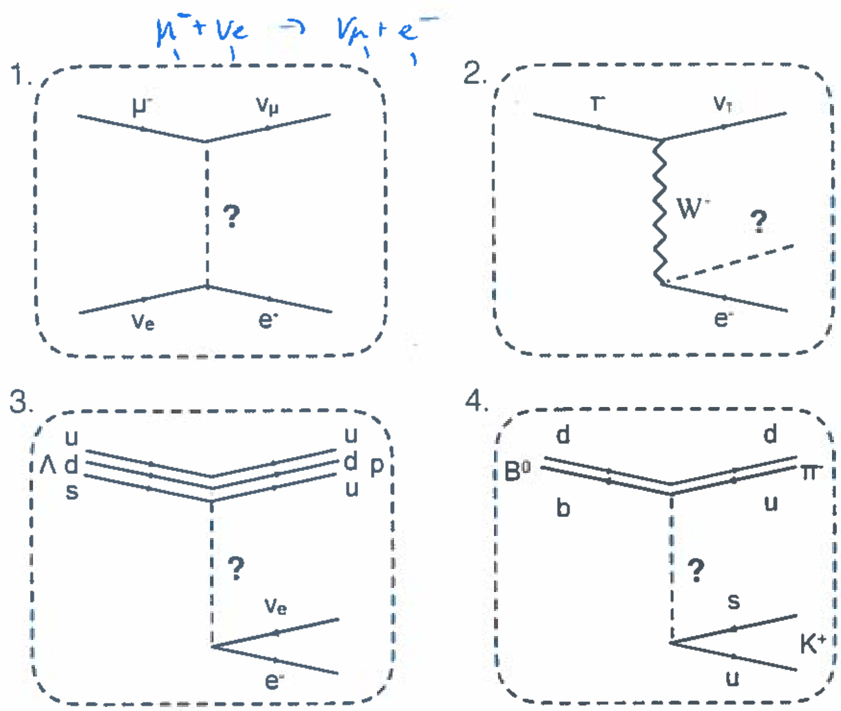
Exercise 4

Particles and quantum numbers, Feynman diagrams and conservation laws

(a) Examine the following processes, indicate if they can or can not take place and in the latter case list all the reasons why. In parenthesis, you can find the quark composition of hadrons, when present. **(2)**

1. $\mu^+ \to e^+ + e^- + e^+ + \bar{\nu}_\mu + \nu_e$
2. $\mathrm{p(uud)} \to e^+ + \nu_e$
3. $\tau^+ + p(\mathrm{uud}) \to e^+ + p(\mathrm{uud}) + \bar{\nu}_\tau$
4. $\pi^0(\mathrm{u\bar{u}\ or\ d\bar{d}}) \to e^+ + e^-$

(b) The following Feynman diagrams are incomplete. Try to complete the missing parts, considering the lowest order contributions. **(2)**



(c) Identify the type of interaction (i.e. electromagnetic, weak or strong) and write down the mediator. In parenthesis, you can find the quark composition of hadrons, when present. **(2)**

1. $\Lambda(\mathrm{uds}) \to \mathrm{p(uud)} + \pi^-(\mathrm{d\bar{u}})$
2. $e^+ + e^- \to \tau^+ + \tau^-$
3. $K_S^0(\mathrm{d\bar{s}}) \to \pi^+(\mathrm{u\bar{d}}) + \pi^-(\mathrm{d\bar{u}})$
4. $\mathrm{p} + \bar{\nu}_\mu \to \mathrm{p} + \bar{\nu}_\mu$