Structure of Matter (NS-266B)

Exam

14 April 2014; time: 3 hours

The exam consists of two parts: Exercises on Condensed Matter (Part I) and Subatomic Physics (Part II).

The maximal number of points is indicated for each exercise.
The total number of points is 103.

→ Answer each of the exercises on a separate piece of paper.
→ Write your name and student number on each page.
→ Do not give final answers only, explain your reasoning (short) and/or give full calculations.
→ Calculator use is allowed (no cell/smart phone!).

Success!

Part I – Condensed Matter

Exercise 1.1 (25 points)

For the following situations make a 1D or 2D graph in which you explain where and what the features are that are visible:

a) (5 points) The radial distribution function for an ideal gas composed of hard spheres with a diameter of 1 nm.

b) (5 points) The radial distribution function for a liquid that is at such a low concentration that only two spherical atoms interact at the same time. The way the particles interact is through a so-called hard-sphere Yukawa potential which is a purely repulsive potential with a range of about three times the diameter of the hard core, which is 1 nm.

c) (5 points) The radial distribution function of a 2D hexagonal close packed crystal of hard spheres with a diameter of 1 nm (you need to draw this function only for $r < 5$ nm).

d) (5 points) The 2D scattering pattern for a nematic liquid crystal phase with its director perfectly aligned in the plane parallel to the scattering plane. The dimensions of the stiff rod-like molecules are: diameter 1 nm, length of rod: 10 nm.

e) (5 points) The 2D scattering pattern for a nematic liquid crystal phase with its director perfectly aligned perpendicular to the scattering plane. The dimensions of the stiff rod-like molecules are again: diameter 1 nm, length of rod: 10 nm.

Exercise 1.2 (20 points)

An important way to characterize the conduction of metals is by using the Hall Effect which is evaluated by applying a known voltage which gives rise to an electric field E over the length of a metal bar that is placed within a known electric field B as indicated in the following figure:

The equation of motion for an electron with a velocity v is given by the general relation:

$$\vec{F} = \frac{d\vec{p}}{dt} = -e\left(\vec{E} + \vec{v} \times \vec{B}\right) - \frac{\vec{p}}{\tau_{coll}}$$

in which $-p/\tau_{coll}$ is the effective drag force on the electron (expressed using the average collision time τ_{coll}).

a) (8 points) Derive the two simultaneous equations that apply for the motion of the electron in the horizontal plane of the bar.

b) (8 points) Use steady state conditions (accelerations and velocity of the electron in the y direction have become 0) and the following relation between the current density J and n_e , the density of the mobile valence electrons (and the mass and charge of the electron: m, e):

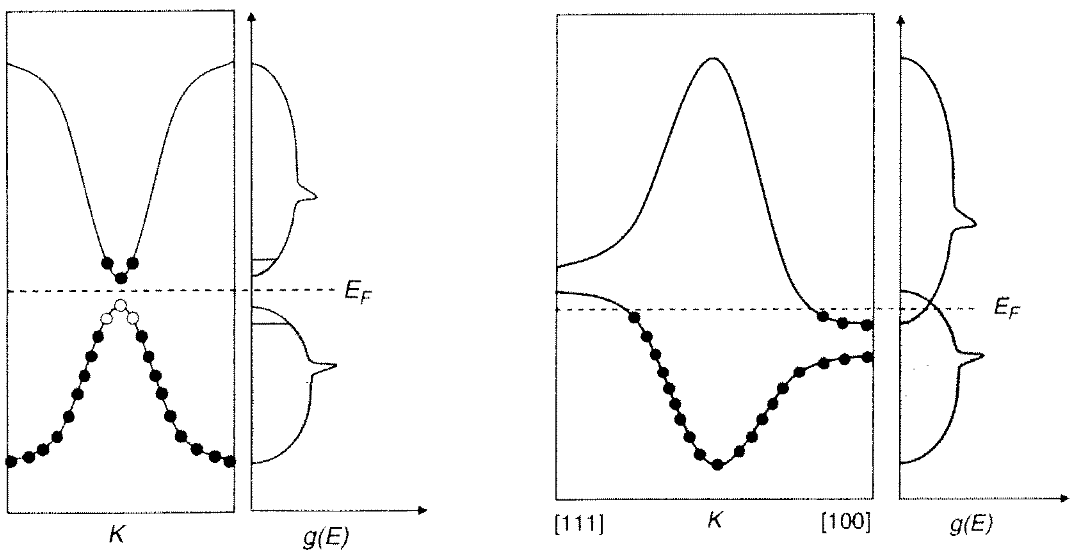
$$J = \left(\frac{n_e e^2 \tau_{col}}{m}\right) E$$

to prove that the Hall coefficient R_H, which is defined by $R_H \equiv E_y/J_x B$ is equal to $= -1/n_e e$.

c) (4 points) Explain how it is possible that measurements of the Hall coefficient for certain metals (like Al or In) indicate a positive charge of the valence electrons and why this cannot be explained by the (almost) free electron model of conductivity.

Exercise 1.3 (20 points)

Many semiconductors are characterized by the band diagram and density of states plot as given in the following left figure:



a) (8 points) Contrary to the behavior of the conduction of metals when the temperature for such semiconductors is raised the conductivity strongly rises instead of falling as it does for metals. What relationship from statistical mechanics do you need to explain this behavior? And use it to explain this strong decrease in resistivity with increasing temperature.

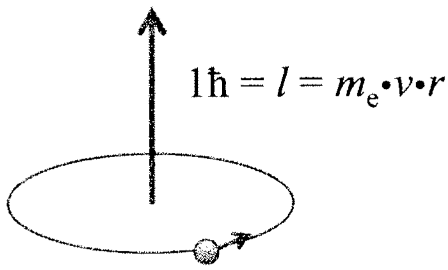
b) (8 points) Mention several other mechanisms that can enhance the conductivity of semiconductors with the band diagram given in the left figure without raising the temperature and explain how they increase the conductivity. Which is presently the most important for the semiconductor industry (explain why)?

c) (4 points) For some divalent crystals the band diagram and density of states plot is as given in the Right Figure. Explain by using the mechanism behind the technique of scanning tunneling microscopy (STM) if you expect that it is possible to image surface atoms in a solid characterized by the band diagram on the right or not (provide also the reasoning behind your answer).

Part II – Subatomic Physics

Exercise 2.1: Hyperfine structure (10 points)

a) (6 points) Estimate the magnetic field generated by the lowest electron shell at the site of the atomic nucleus in the following way:



- Ring current I by orbiting electron
- Orbital angular momentum is $1\hbar$
- Orbital radius r = Bohr radius
- Magnetic field in the center of a coil

$B = \mu_0 \cdot n \cdot I/l$ with $n = 1$ and $l = 2r_B$
[1 Tesla = 1 Vs/m^2]

b) (2 points) Calculate the nuclear magneton $\mu_k = e \cdot \hbar/(2m_p)$ [in eV/T = e·m^2/s].

c) (2 points) Calculate from a) and b) the typical hyperfine splitting $\Delta E = \mu_k \cdot B$. In case you have not solved a) assume a magentic field of 12,5 Tesla.
What is the frequency of the hyperfine transistion?

Useful parameters:

$r_B = 5{,}2918 \cdot 10^{-11} m$

$\mu_0 = 4\pi \cdot 10^{-7} \ Vs/Am$

$\hbar c = 197{,}33 \ MeV \cdot fm$

$h = 6{,}626 \cdot 10^{-34} Js$

$c = 3 \cdot 10^8 \ m/s$

$e = 1{,}602 \cdot 10^{-19} As$

$m_{electron} = 511 \ keV/c^2$

$m_{protron} = 938 \ MeV/c^2$

Exercise 2.2: Conservation laws (16 points)

a) (2 points each) Check the conservation laws (mass, electric charge, strangeness and baryon number) for the following reactions/decays and say whether they are allowed or forbidden:

A) $p \rightarrow e^+ + \pi^0$

B) $e^- \rightarrow \nu_e + \gamma$

C) $\Lambda^0 \rightarrow p + K^-$

D) $\pi^+ + p \rightarrow \Delta^{++} + \pi^0$

E) $\Xi^- + p \rightarrow \Omega^- + K^+ + n$

b) (3 points each) Write down the quark content of each particle for the reaction (see enclosed tables) and the Feynman diagram on quark level:

F) $\Lambda^0 \rightarrow p + \pi^-$

G) $K^- + p \rightarrow \Xi^0 + K^0$

Exercise 2.3: Particle production and identification (12 points)

A liquid hydrogen target is bombarded with a proton beam of a momentum of p = 12 GeV/c. The momentum of the reaction products (particles) are measured in wire chambers inside a magnetic field. In one event six charged particle tracks are seen. Two of them go back to the interaction vertex. They belong to positively charged particles. The other tracks come from two pairs of oppositely charged particles. Each of these pairs appears a few centimeters away from the interaction point, indicating the decay of strange particles. Evidently two electrical neutral, and hence unobservable, particles were created, which later both decayed into pairs of charged particles.
The reaction was reconstructed as: $p + p \rightarrow X_1 + X_2 + p + \pi^+ \rightarrow p + \pi^- + \pi^+ + \pi^- + p + \pi^+$, where X_1 and X_2 are the unknown neutral particles.

a) (2 points) Use table 12-3 (showing typical particle decays) to discuss which mesons and baryons could be responsible for the two observed decays.

b) (2 points) Make a rough sketch of the tracks of the reaction products.

c) (5 points) The measured momenta of the decay pairs were:

1) $|\vec{p}^+| = 0{,}68\ GeV/c,\ |\vec{p}^-| = 0{,}27\ GeV/c,\ \angle(\vec{p}^+, \vec{p}^-) = 11^\circ$

2) $|\vec{p}^+| = 0{,}25\ GeV/c,\ |\vec{p}^-| = 2{,}16\ GeV/c,\ \angle(\vec{p}^+, \vec{p}^-) = 16^\circ$

Use the *invariant mass* method

$$m_{inv}c^2 = \sqrt{(E_1 + E_2)^2 - (\vec{p}_1 c + \vec{p}_2 c)^2}$$

to reconstruct the neutral particles, which decayed into the observed charged particles with energy

$$E = \sqrt{(|\vec{p}|c)^2 + (mc^2)^2}$$

Check whether your hypotheses from a) are compatible with these results.

d) (3 points) Using these results and considering all conservation laws give a scheme for the total reaction (indicating the unknown neutral particles).
Is the solution unique?

Table 12-3 Hadrons that are stable against decay via the strong interaction

Name	Symbol	Mass (MeV/c^2)	Spin ($\hbar$)	Charge (e)	Antiparticle	Mean lifetime (s)	Typical decay products*
Baryons							
Nucleon	p (proton) or N^+	938.3	1/2	+1	$\bar{p}$	$>10^{32}$ y	
	n (neutron) or N^0	939.6	1/2	0	$\bar{n}$	930	$p + e^- + \bar{\nu}_e$
Lambda	Λ^0	1116	1/2	0	$\bar{\Lambda}^0$	2.5×10^{-10}	$p + \pi^-$
Sigma	Σ^+	1189	1/2	+1	$\bar{\Sigma}^-$	0.8×10^{-10}	$n + \pi^+$
	Σ^0	1192	1/2	0	$\bar{\Sigma}^0$	10^{-20}	$\Lambda^0 + \gamma$
	Σ^-	1197	1/2	−1	$\bar{\Sigma}^+$	1.7×10^{-10}	$n + \pi^-$
Xi†	Ξ^0	1315	1/2	0	$\bar{\Xi}^0$	3.0×10^{-10}	$\Lambda^0 + \pi^0$
	Ξ^-	1321	1/2	−1	$\bar{\Xi}^+$	1.7×10^{-10}	$\Lambda^0 + \pi^-$
Omega	Ω^-	1672	3/2	−1	Ω^+	1.3×10^{-10}	$\Xi^0 + \pi^-$
Charmed lambda	Λ_c^+	2285	1/2	+1	$\bar{\Lambda}_c^-$	1.8×10^{-13}	$p + K^- + \Lambda^+$
Mesons							
Pion	π^+	139.6	0	+1	π^-	2.6×10^{-8}	$\mu^+ + \nu_\mu$
	π^0	135	0	0	self	0.8×10^{-16}	$\gamma + \gamma$
	π^-	139.6	0	−1	π^+	2.6×10^{-8}	$\mu^- + \bar{\nu}_\mu$
Kaon	K^+	493.7	0	+1	K^-	1.24×10^{-8}	$\pi^+ + \pi^0$
	K^0	497.7	0	0	$\bar{K}^0$	0.88×10^{-10}	$\pi^+ + \pi^-$
						and	
						5.2×10^{-8}‡	$\pi^+ + e^- + \bar{\nu}_e$
Eta	η^0	549	0	0	self	2×10^{-19}	$\gamma + \gamma$

*Other decay modes also occur for most particles.
†The Ξ particle is sometimes called the cascade.
‡The K^0 has two distinct lifetimes, sometimes referred to as K^0_{short} and K^0_{long}. All other particles have a unique lifetime.

Table 12-11 Quark composition of selected hadrons

Baryons	Quarks	Mesons	Quarks
p	uud	π^+	$u\bar{d}$
n	udd	π^-	$\bar{u}d$
Λ^0	uds	K^+	$u\bar{s}$
Δ^{++}	uuu	K^0	$d\bar{s}$
Σ^+	uus	$\bar{K}^0$	$s\bar{d}$
Σ^0	uds	K^-	$s\bar{u}$
Σ^-	dds	J/ψ	$c\bar{c}$
Ξ^0	uss	D^+	$c\bar{d}$
Ξ^-	dss	D^0	$c\bar{u}$
Ω^-	sss	D_s^+	$c\bar{s}$
Λ_c^+	udc	B^+	$u\bar{b}$
Σ_c^{++}	uuc	$\bar{B}^0$	$\bar{d}b$
Σ_c^+	udc	B^0	$d\bar{b}$
Ξ_c^+	usc	B^-	$\bar{u}b$

Table 12-6 Some quantum numbers of the hadrons that are stable against decay via the strong interaction

Particle	Spin, $\hbar$	I	I_3	B	S	Y
p	1/2	1/2	+1/2	1	0	1
n	1/2	1/2	−1/2	1	0	1
Λ^0	1/2	0	0	1	−1	0
Σ^+	1/2	1	+1	1	−1	0
Σ^0	1/2	1	0	1	−1	0
Σ^-	1/2	1	−1	1	−1	0
Ξ^0	1/2	1/2	+1/2	1	−2	−1
Ξ^-	1/2	1/2	−1/2	1	−2	−1
Ω^-	3/2	0	0	1	−3	−2
π^+	0	1	+1	0	0	0
π^0	0	1	0	0	0	0
π^-	0	1	−1	0	0	0
K^+	0	1/2	+1/2	0	+1	+1
K^0	0	1/2	−1/2	0	+1	+1
η^0	0	0	0	0	0	0