Niels Tuning* and Panos Christakoglou†

Particle Physics II

Retake Exam (Every sub-question has the same absolute weight.)

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*h71@nikhef.nl - Nikhef, Science Park 105, 1098XG, Amsterdam, The Netherlands
†Panos.Christakoglou@nikhef.nl - Nikhef, Science Park 105, 1098XG, Amsterdam, The Netherlands

Exercise 1

Fermion masses the Standard Model and Flavour Mixing

In the Standard Model, the Yukawa couplings give rise to the following couplings between different generations:

$$
\begin{aligned}
-\mathcal{L}_{Yukawa} &= Y_{ij}\overline{\psi_{Li}}\,\phi\,\psi_{Rj} + h.c. \\
&= Y^{d}_{ij}\overline{Q^{I}_{Li}}\,\phi\,d^{I}_{Rj} + Y^{u}_{ij}\overline{Q^{I}_{Li}}\,\tilde{\phi}\,u^{I}_{Rj} + Y^{l}_{ij}\overline{L^{I}_{Li}}\,\phi\,l^{I}_{Rj} + h.c.
\end{aligned} \tag{0.0.1}
$$

(a) Which of these three terms hides the mass terms for electrons? What is the value of the spin of the Higgs? To which bosons does the right-handed electron couple in the SM ? To which bosons does the left-handed electron couple in the SM ?

(b) What do the above Yukawa couplings tell us about the mass of the neutrino? To which bosons does the right-handed neutrino couple in the SM ?

(c) The observation of neutrino oscillations implies that neutrinos have non-zero mass. How would you add neutrino masses to the Standard Model? To which bosons does the right-handed neutrino couple in your extension of the Standard Model ? Would you now call the right-handed neutrino 'sterile' ?

(d) The lepton-equivalent of the CKM-matrix is called the MNSP-matrix.

$$
U_{MNSP}: \begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}| \end{pmatrix} \approx \begin{pmatrix} 0.85 & 0.53 & 0 \\ 0.37 & 0.60 & 0.71 \\ 0.37 & 0.60 & 0.71 \end{pmatrix}.
$$

Compare the coupling strength of an electron to the various mass-eigenstates of the neutrino: what is the fraction of ν_1 in the process $e^- \to W^- \nu$? What is the fraction of ν_e in the process $e^- \to W^- \nu$?

(e) Only the magnitude of the mixing matrix elements are shown above. In order to have CP-violation in the neutrino sector, there must be a non-vanishing relative complex phase between matrix elements. To measure CP-violation in the neutrino sector, which of the three categories of CP-violation ("in decay", "in mixing" and/or "in the interference between decay and mixing+decay") is the best strategy? Explain why.

Exercise 2

A CP measurement with the decays $B^+ \to D^0 K^+$

We will have a look at decays of charged B mesons.

(a) Which of the three categories of CP-violation ("in decay", "in mixing" and/or "in the interference between decay and mixing+decay") apply to CP-violation in decays of charged B mesons? Explain why.

(b) Draw the Feynman diagram of the decay $B^+ \to \bar{D}^0 K^+$ (I). Indicate the appropriate CKM-elements at the vertices. (NB: A $\bar{D}^0$ meson consists of a $\bar{c}$ and a u quark.)

(c) Draw the Feynman diagram of the decay $B^+ \to D^0 K^+$ (II). Which of the above two decay modes (I) or (II) is colour-suppressed? Explain. Do the above two decay modes (I) and (II) interfere? Explain why (not).

(d) Give an example of a D^0 decay to a CP-eigenstate.

(e) Do you expect to observe CP violation in decays of charged B mesons to a neutral D meson and a charged kaon? Why?

(f) What is the relative weak phase difference between the two decays (I) and (II)?

(g) What D final state would you choose to study CP violation? Why?

(h) To which CP violating parameter(s) is this measurement sensitive?

(i) Would you expect CP violation effects in $B^+ \to D^0\pi^+$ and $B^+ \to \bar{D}^0\pi^+$ decays? Explain why (not). p. 53

Exercise 3

Group theory, form factors and the colour quantum number

(a)Confirm that the generators of SO(3) satisfy the algebra $[\hat{X}_i, \hat{X}_j] = i\varepsilon_{ijk}\hat{X}_k$. One combination of i, j, k is enough.

(b)Consider the operator $\hat{U} = e^{-i\varepsilon_3\hat{F}/\hbar}$, where ε_3 is a rotation in real space around the z axis and $\hat{F}$ the generator of the transformation.

1. Identify mathematically the symmetry generator of the transformation if the Hamiltonian of the system is invariant under translation in space.
2. Identify which is the conserved quantity?

(c)Two particles, each of spin 2 and third component 0, form a composite system whose orbital angular momentum is 0.

1. What is the probability for each of the states of the composite system?
2. Which state is the most probable?
3. Show that the probabilities add up to unity.

(d)One of the well known QED processes is the elastic scattering of energetic electrons off protons at rest. The cross section of such process is given at the end by the so-call Rosenbluth formula.

1. Write down its expression and explain what the G_E and G_M-terms physically represent.
2. What do we learn from them?

(e)Considering the quark contents of known hadrons, argue why there was a need to introduce a new quantum number i.e. the one of colour.

(f) Observe the plot given in fig. 0.1.

1. Describe what the plot shows (i.e. what is plotted on both axis),
2. discuss what it reveals.
3. Explain why the data points have a systematic shift at given positions of Q.

Exercise 4

Gluons, the strong coupling constant, the QCD Lagrangian and heavy-ions

(a)There are eight physical gluon species which are described by the SU(3) color symmetry.

1. Write down the different colour states of gluons.
2. How do the gluons manifest their existence in elementary processes such as $e^+ + e^-$?

(b)In the previous exercise, you were asked to write down the eight physical gluon states. Identify which of these gluon states are mediating the following interactions between (anti)quarks, shown in fig. 0.2.

(c)Calculate the colour factors for the following state transition $B\bar{G} \to B\bar{G}$

(d)Show that the colour interactions between a quark and an antiquark are attractive in the colour singlet state.

(e)The QCD Lagrangian in the Standard Model is given by

$$\mathscr{L}_{\mathrm{QCD}} = \overline{\psi}(i\partial\!\!\!/ - m)\psi - \tfrac{1}{4}F^{\mu\nu}\cdot F_{\mu\nu} - (g_s\overline{\psi}\gamma^\mu\lambda\psi)\cdot A_\mu + \cdots$$

1. Which part includes the self coupling terms of the gauge bosons of the theory?

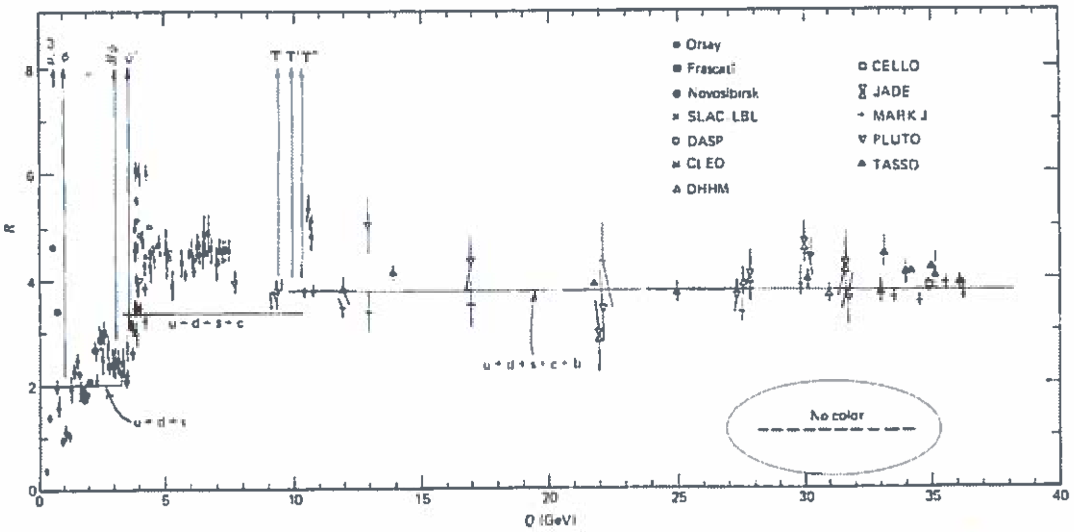

Fig. 11.3 Ratio R of (11.6) as a function of the total $e^- e^+$ center-of-mass energy. (The sharp peaks correspond to the production of narrow 1 resonances just below or near the flavor thresholds.)

Fig. 0.1

2. What kind of particles do the ψ and $\overline{\psi}$ spinors represent?
3. Which is the term that describes the interaction between the particles and the fields?
4. Which term gives the strength of the interaction and how does it depend on the "hardness" of the process?

(f) There are undisputed evidence that the quark gluon plasma (QGP) has been created in ultra-relativistic heavy-ion collisions at both the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC).

1. Describe briefly what the QGP is.
2. Argue why its existence does not contradict confinement?

For these you have to consider that the colour matrices are given by:

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ for R, } \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ for B, } \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ for G}$$

The Gell-Mann matrices are:

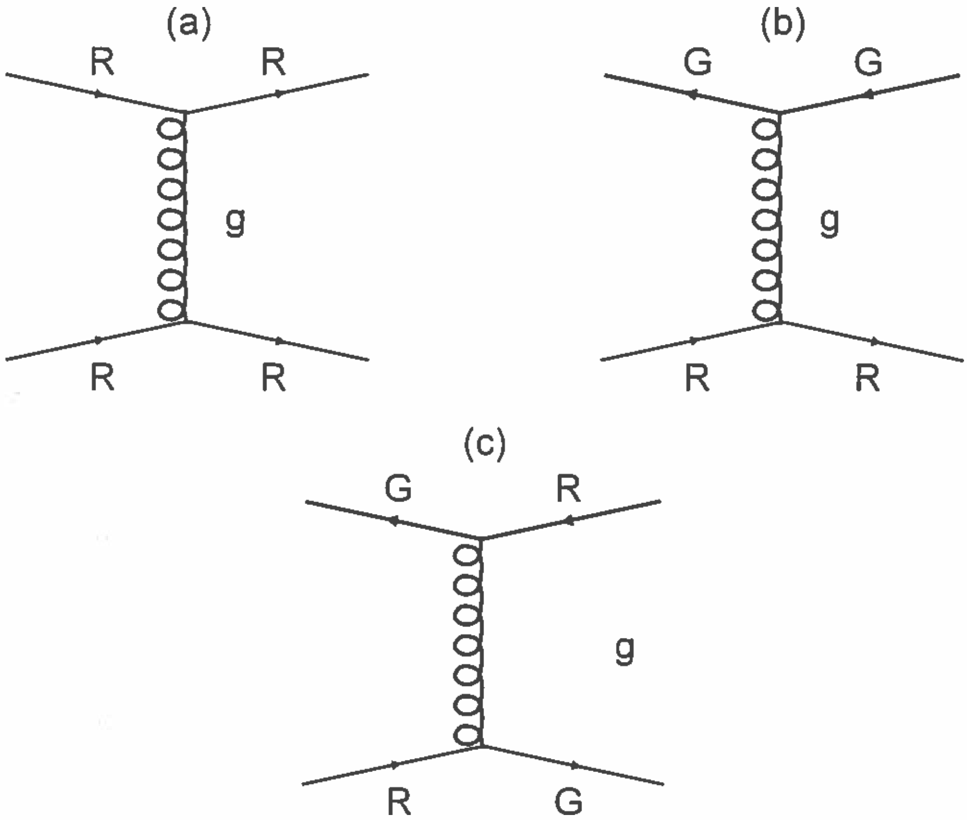

Fig. 0.2: Different QCD processes

$$\underbrace{\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}}_{\lambda_1}\quad\underbrace{\begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix}}_{\lambda_2}\quad\underbrace{\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}}_{\lambda_3}\quad\underbrace{\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix}}_{\lambda_4}$$

$$\underbrace{\begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix}}_{\lambda_5}\quad\underbrace{\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}}_{\lambda_6}\quad\underbrace{\begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix}}_{\lambda_7}\quad\frac{1}{\sqrt{3}}\underbrace{\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}}_{\lambda_8}$$

36. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

m_1	m_2	J / M	J / M	...
m_1	m_2	Coefficients		
⋮	⋮			

1/2×1/2

m_1	m_2	M	$J=1$	$J=0$
+1/2	+1/2	+1	1	
+1/2	−1/2	0	1/2	1/2
−1/2	+1/2	0	1/2	−1/2
−1/2	−1/2	−1	1	

1×1/2

m_1	m_2	M	$J=3/2$	$J=1/2$
+1	+1/2	+3/2	1	
+1	−1/2	+1/2	1/3	2/3
0	+1/2	+1/2	2/3	−1/3
0	−1/2	−1/2	2/3	1/3
−1	+1/2	−1/2	1/3	−2/3
−1	−1/2	−3/2	1	

2×1

m_1	m_2	M	$J=3$	$J=2$	$J=1$
+2	+1	+3	1		
+2	0	+2	1/3	2/3	
+1	+1	+2	2/3	−1/3	
+2	−1	+1	1/15	1/3	3/5
+1	0	+1	8/15	1/6	−3/10
0	+1	+1	2/5	−1/2	1/10
+1	−1	0	1/5	1/2	3/10
0	0	0	3/5	0	−2/5
−1	+1	0	1/5	−1/2	3/10
0	−1	−1	2/5	1/2	1/10
−1	0	−1	8/15	−1/6	−3/10
−2	+1	−1	1/15	−1/3	3/5
−1	−1	−2	2/3	1/3	
−2	0	−2	1/3	−2/3	
−2	−1	−3	1		

1×1

m_1	m_2	M	$J=2$	$J=1$	$J=0$
+1	+1	+2	1		
+1	0	+1	1/2	1/2	
0	+1	+1	1/2	−1/2	
+1	−1	0	1/6	1/2	1/3
0	0	0	2/3	0	−1/3
−1	+1	0	1/6	−1/2	1/3
0	−1	−1	1/2	1/2	
−1	0	−1	1/2	−1/2	
−1	−1	−2	1		

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

2×1/2

m_1	m_2	M	$J=5/2$	$J=3/2$
+2	+1/2	+5/2	1	
+2	−1/2	+3/2	1/5	4/5
+1	+1/2	+3/2	4/5	−1/5
+1	−1/2	+1/2	2/5	3/5
0	+1/2	+1/2	3/5	−2/5
0	−1/2	−1/2	3/5	2/5
−1	+1/2	−1/2	2/5	−3/5
−1	−1/2	−3/2	4/5	1/5
−2	+1/2	−3/2	1/5	−4/5
−2	−1/2	−5/2	1	

3/2×1/2

m_1	m_2	M	$J=2$	$J=1$
+3/2	+1/2	+2	1	
+3/2	−1/2	+1	1/4	3/4
+1/2	+1/2	+1	3/4	−1/4
+1/2	−1/2	0	1/2	1/2
−1/2	+1/2	0	1/2	−1/2
−1/2	−1/2	−1	3/4	1/4
−3/2	+1/2	−1	1/4	−3/4
−3/2	−1/2	−2	1	

3/2×1

m_1	m_2	M	$J=5/2$	$J=3/2$	$J=1/2$
+3/2	+1	+5/2	1		
+3/2	0	+3/2	2/5	3/5	
+1/2	+1	+3/2	3/5	−2/5	
+3/2	−1	+1/2	1/10	2/5	1/2
+1/2	0	+1/2	3/5	1/15	−1/3
−1/2	+1	+1/2	3/10	−8/15	1/6
+1/2	−1	−1/2	3/10	8/15	1/6
−1/2	0	−1/2	3/5	−1/15	−1/3
−3/2	+1	−1/2	1/10	−2/5	1/2
−1/2	−1	−3/2	3/5	2/5	
−3/2	0	−3/2	2/5	−3/5	
−3/2	−1	−5/2	1		

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

3/2×3/2

m_1	m_2	M	$J=3$	$J=2$	$J=1$	$J=0$
+3/2	+3/2	+3	1			
+3/2	+1/2	+2	1/2	1/2		
+1/2	+3/2	+2	1/2	−1/2		
+3/2	−1/2	+1	1/5	1/2	3/10	
+1/2	+1/2	+1	3/5	0	−2/5	
−1/2	+3/2	+1	1/5	−1/2	3/10	
+3/2	−3/2	0	1/20	1/4	9/20	1/4
+1/2	−1/2	0	9/20	1/4	−1/20	−1/4
−1/2	+1/2	0	9/20	−1/4	−1/20	1/4
−3/2	+3/2	0	1/20	−1/4	9/20	−1/4
+1/2	−3/2	−1	1/5	1/2	3/10	
−1/2	−1/2	−1	3/5	0	−2/5	
−3/2	+1/2	−1	1/5	−1/2	3/10	
−1/2	−3/2	−2	1/2	1/2		
−3/2	−1/2	−2	1/2	−1/2		
−3/2	−3/2	−3	1			

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^1_{1,1} = \frac{1+\cos\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

2×3/2

m_1	m_2	M	$J=7/2$	$J=5/2$	$J=3/2$	$J=1/2$
+2	+3/2	+7/2	1			
+2	+1/2	+5/2	3/7	4/7		
+1	+3/2	+5/2	4/7	−3/7		
+2	−1/2	+3/2	1/7	16/35	2/5	
+1	+1/2	+3/2	4/7	1/35	−2/5	
0	+3/2	+3/2	2/7	−18/35	1/5	
+2	−3/2	+1/2	1/35	6/35	2/5	2/5
+1	−1/2	+1/2	12/35	5/14	0	−3/10
0	+1/2	+1/2	18/35	−3/35	−1/5	1/5
−1	+3/2	+1/2	4/35	−27/70	2/5	−1/10
+1	−3/2	−1/2	4/35	27/70	2/5	1/10
0	−1/2	−1/2	18/35	3/35	−1/5	−1/5
−1	+1/2	−1/2	12/35	−5/14	0	3/10
−2	+3/2	−1/2	1/35	−6/35	2/5	−2/5
0	−3/2	−3/2	2/7	18/35	1/5	
−1	−1/2	−3/2	4/7	−1/35	−2/5	
−2	+1/2	−3/2	1/7	−16/35	2/5	
−1	−3/2	−5/2	4/7	3/7		
−2	−1/2	−5/2	3/7	−4/7		
−2	−3/2	−7/2	1			

2×2

m_1	m_2	M	$J=4$	$J=3$	$J=2$	$J=1$	$J=0$
+2	+2	+4	1				
+2	+1	+3	1/2	1/2			
+1	+2	+3	1/2	−1/2			
+2	0	+2	3/14	1/2	2/7		
+1	+1	+2	4/7	0	−3/7		
0	+2	+2	3/14	−1/2	2/7		
+2	−1	+1	1/14	3/10	3/7	1/5	
+1	0	+1	3/7	1/5	−1/14	−3/10	
0	+1	+1	3/7	−1/5	−1/14	3/10	
−1	+2	+1	1/14	−3/10	3/7	−1/5	
+2	−2	0	1/70	1/10	2/7	2/5	1/5
+1	−1	0	8/35	2/5	1/14	−1/10	−1/5
0	0	0	18/35	0	−2/7	0	1/5
−1	+1	0	8/35	−2/5	1/14	1/10	−1/5
−2	+2	0	1/70	−1/10	2/7	−2/5	1/5
+1	−2	−1	1/14	3/10	3/7	1/5	
0	−1	−1	3/7	1/5	−1/14	−3/10	
−1	0	−1	3/7	−1/5	−1/14	3/10	
−2	+1	−1	1/14	−3/10	3/7	−1/5	
0	−2	−2	3/14	1/2	2/7		
−1	−1	−2	4/7	0	−3/7		
−2	0	−2	3/14	−1/2	2/7		
−1	−2	−3	1/2	1/2			
−2	−1	−3	1/2	−1/2			
−2	−2	−4	1				

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\,\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\,\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2$$

$$d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Figure 36.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).