2

Re-Exam Algorithms and Networks 2012

This is the re-exam for part 2. Make sure you do the right re-exam. It replaces the note for re-exam 2. If you need to do a re-exam for part 1, ask the teacher as soon as possible for the correct exercise set.

You have three hours for the exam. You may give your answers in Dutch or in English. Write clearly. You may consult four sides of A4 with notes.

Results used in the course or exercise sets may be used without further proof, unless explicitly asked.

Each of the six questions counts for the same number of points.

Some parts are harder than others: use your time well, and make sure you first finish the easier parts!

Good luck!

1. Basic knowledge For each of the following statements, tell if these are *true* or *false* if $P \neq NP$. Give a *very short* explanation of your anwers. (At most one or two lines per item.)

1. There is a polynomial time algorithm for the Travelling Salesman Problem.
2. There is a polynomial time algorithm for the following problem:

   Given: Undirected graph $G = (V, E)$
   Question: Is G planar?

3. The following problem belongs to the class NP:

   Given: Undirected graph $G = (V, E)$
   Question: Is G planar?

4. All problems in NP are NP-complete.

2. Number of independent sets on paths and cycles Show that the following problem can be solved in polynomial time.

Given: Undirected graph $G = (V, E)$ such that each vertex in G has at degree at most two.
Question: What is the number of independent sets in G?

3. Exponential time algorithms Give an algorithm that solves the following problem in $O^*(1.4423^n)$ time:

Given: Undirected graph G, ~~integer K~~
Question: What is the number of independent sets in G ?

Explain your answer!

1

4. Bandwidth and Treewidth An undirected graph $G = (V, E)$ is said to have *bandwidth at most* k, if there exists a numbering of the vertices f (i.e., f is a bijection $V \to \{1, 2, \ldots, |V|\}$), such that for all edges $\{v, w\} \in E$, $|f(v) - f(w)| \leq k$.

Show that if an undirected graph G has bandwidth at most k, then G has treewidth at most k.

5. Kernelization: distance from a clique Consider the following problem.

> **Given:** Undirected graph $G = (V, E)$, integer K
> **Question:** Is there a set of at most K vertices, $W \subseteq V$, such that if we remove W, we obtain a clique.

Show that this problem has a kernel.

6. NP-completeness proof: TSP with distances 2 and 3 Show that the following problem is NP-complete.

> **Given:** Set of n cities $v_1, \ldots, v_n$, and for each pair of different cities v_i, v_j, $i \neq j$, a distance $d(v_i, v_j) \in \{2, 3\}$, integer K. **Question:** Does there exist a TSP-tour such that the total length of the tour is at most K?

I.e., this is the special case of TSP where 2 and 3 are the only possible values for distances.